

Creation of the universe with a stealth scalar field

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The stealth scalar field is a non-trivial configuration without any back-reaction to geometry, which is characteristic for non-minimally coupled scalar fields. Studying the creation probability of the de Sitter universe with a stealth scalar field by the Hartle and Hawking's semi-classical method, we show that the effect of the stealth field can be significant. For the class of scalar fields we consider, creation of the universe with a stealth field is possible for a discrete value of the coupling constant and its creation rate is almost the same as that of the universe in vacuum.

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I. INTRODUCTION

At present, the framework of the Big-Bang cosmology with an early inflationary era achieves a clear consensus from cosmologists. Although the number of the first observational data by Hubble advocating the expanding universe was small [1], a variety of current observations supports the assertion. On the other hand, another fact that the universe is spatially homogeneous and isotropic in a large scale indicates that the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological spacetime may be a good model as the zeroth-order approximation of our universe. A direct consequence of these two is that there was an extremely dense and hot era in the history of our universe, called the Big Bang.

While the great success of the Big-Bang cosmology is the prediction of the observed cosmic microwave background (CMB) and the production of the light elements, several problems are left unsolved in this framework. Among them, the flatness problem, horizon problem, and monopole problem (if one believes grand unified theories of fundamental interactions) are resolved by an additional but simple assumption; there was a phase with accelerating expansion before the Big-Bang era. This assumption of the cosmic inflation suits the observational data of the CMB quite well and completes the modern framework of the Big-Bang cosmology.

Even in this framework, however, the initial singularity problem remains unsolved and clearly shows the limit of the classical description of the early universe. The Big-Bang singularity is a consequence of the singularity theorems but the strong energy condition is assumed to prove them in general relativity [2]. By this reason, one may think that inflation is able to slip through the net of the singularity theorems. However, under certain reasonable conditions without energy conditions, it was shown that the inflationary universe must have a past spacetime boundary, namely the initial moment of the classical universe [3].

It is reasonable to assume that the quantum aspect of gravity dominates close to the initial singularity. This leads us to quantum cosmology, which is a branch of the minisuperspace approach of canonical quantum gravity [4]. In quantum cosmology, the Wheeler-de Witt equation, obtained by the quantization of the Hamiltonian constraint of Einstein equations, determines the wave function of the universe. In this context, Vilenkin proposed the spontaneous creation of the universe by quantum tunneling from nothing [5, 6]. This picture is understood as the creation of the universe in the de Sitter space, represented by the Euclidean solution called instanton. Because the mathematical description is analogous to the quantum tunneling through the potential barrier, it is claimed that the creation probability is proportional to $e^{-|S_E|/\hbar}$ [7], where S_E is the value of the Euclidean action evaluated for the instanton solution. On the other hand, Hartle and Hawking made the no-boundary proposal claiming that the wave function of the universe is obtained by a path integral over non-singular compact Euclidean spaces [8, 9], by which they evaluated the creation probability as $e^{-S_E/\hbar}$.

The simplest vacuum instanton solution is the Hawking-Moss de Sitter instanton which is nothing but an S^4 in the 4D Euclidean space and can be analytically continued to the Lorentzian de Sitter universe with the spatially closed slicing [10]. While the value of the Euclidean action in vacuum is simply proportional to the volume of the instanton space, the contribution of matter to the creation probability is non-trivial. Non-minimally coupled scalar fields are interesting in this context because they allow a “stealth” configuration, namely a non-trivial solution which does not give any back-reaction to gravity. Such a configurations was already recognized in 1976 at the latest both for a conformally coupled scalar field and a Yang-Mills field in the Minkowski spacetime [11]. More recently, the stealth solution was found in the three-dimensional (locally) anti-de Sitter (AdS) spacetime [12] and also in the Minkowski [13], de Sitter (dS) [14], and AdS spacetimes [15] in four dimensions. (See also [16, 17].) Then a natural question arises: if a de Sitter instanton is possible both with a trivial scalar field and with a stealth scalar field, which universe is preferred at the moment of creation? In this paper, we

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consider this problem.

In the following section, we present our system and obtain a stealth solution in the de Sitter instanton space. In Section III, we study the creation probability of the de Sitter universe based on our solutions. Concluding remarks and discussions are summarized in Section IV. Our basic notation follows [18]. The conventions for curvature tensors are $[\nabla_\rho, \nabla_\sigma]V^\mu = R^\mu{}_{\nu\rho\sigma}V^\nu$ and $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$. The signature of the Minkowski spacetime is $(-, +, +, +)$ and Greek indices run over all spacetime indices. We adopt the units such that $c = \hbar = 8\pi G = 1$.

II. STEALTH SCALAR FIELD IN DE SITTER SPACE

A. System

We consider general relativity coupled with a scalar field non-minimally, of which action is given by

$$S = S_g + S_\phi, \quad (2.1)$$

where

$$S_g = \frac{1}{2} \int d^4x \sqrt{-g} (R - 2\Lambda), \quad (2.2)$$

$$S_\phi = - \int d^4x \sqrt{-g} \left[\frac{1}{2} (\nabla\phi)^2 + \frac{1}{2} \xi R \phi^2 + V(\phi) \right]. \quad (2.3)$$

S_ϕ is the action for a non-minimally coupled scalar field. The resulting field equations are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}^{(\phi)}, \quad (2.4)$$

$$\nabla^2 \phi - \xi R \phi - \frac{dV}{d\phi} = 0, \quad (2.5)$$

where

$$T_{\mu\nu}^{(\phi)} = (1 - 2\xi)(\nabla_\mu \phi)(\nabla_\nu \phi) + \left(2\xi - \frac{1}{2} \right) (\nabla\phi)^2 g_{\mu\nu} - V g_{\mu\nu} + \xi \phi^2 G_{\mu\nu} + 2\xi \phi (-\nabla_\mu \nabla_\nu \phi + g_{\mu\nu} \nabla^2 \phi). \quad (2.6)$$

For a conformally coupled scalar field, the coupling constant ξ and the potential are chosen as $\xi = 1/6$ and $V(\phi) = \alpha\phi^4$, where α is a constant, and then the trace of $T_{\mu\nu}^{(\phi)}$ is vanishing.

B. Stealth scalar-field solution

Here we consider the Euclidean de Sitter space:

$$ds^2 = d\tau^2 + a(\tau)^2 \left(d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2) \right), \quad (2.7)$$

$$a(\tau) = \frac{1}{H} \cos H\tau, \quad (2.8)$$

where $H := \sqrt{\Lambda/3}$ and the domain of τ is $-\pi/(2H) \leq \tau \leq \pi/(2H)$. The domains of other coordinates are $0 \leq$

$\chi \leq \pi$, $0 \leq \theta \leq \pi$, and $0 \leq \varphi \leq 2\pi$. We obtain a stealth scalar field in this spacetime, namely a non-trivial scalar field with $T_{\mu\nu}^{(\phi)} \equiv 0$. (The AdS instanton with a stealth conformally coupled scalar field was obtained in [17].) The field equations for the stealth scalar field are

$$0 = (1 - 2\xi)(\nabla_\mu \phi)(\nabla_\nu \phi) + \left(2\xi - \frac{1}{2} \right) (\nabla\phi)^2 g_{\mu\nu} - V g_{\mu\nu} - \xi \Lambda \phi^2 g_{\mu\nu} + 2\xi \phi (-\nabla_\mu \nabla_\nu \phi + g_{\mu\nu} \nabla^2 \phi), \quad (2.9)$$

$$0 = \nabla^2 \phi - 4\xi \Lambda \phi - \frac{dV}{d\phi}, \quad (2.10)$$

where we have used the Einstein equations with vanishing energy-momentum tensor: $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$ and $R = 4\Lambda$.

Assuming $\phi = \phi(\tau)$, we find basic equations:

$$\frac{1}{2}(\phi')^2 - V - \xi \Lambda \phi^2 - 6\xi H \tan H\tau \phi \phi' = 0, \quad (2.11)$$

$$(4\xi - 1)(\phi')^2 - 2V - 2\xi \Lambda \phi^2 + 4\xi \phi (\phi'' - 2H \tan H\tau \phi') = 0, \quad (2.12)$$

$$\phi'' - 3H \tan H\tau \phi' - 4\xi \Lambda \phi - \frac{dV}{d\phi} = 0. \quad (2.13)$$

From Eqs. (2.11) and (2.12), we find

$$(2\xi - 1)(\phi')^2 + 2\xi \phi (\phi'' + H \tan H\tau \phi') = 0, \quad (2.14)$$

from which, we obtain a general solution for the case of $\xi \neq 1/4$:

$$\phi = (p \sin H\tau + q)^{2\xi/(4\xi-1)}, \quad (2.15)$$

where p and q are constants. Inserting the solution (2.15) into (2.11), we find the corresponding potential V as

$$V(\phi) = \frac{\Lambda}{3} \phi^2 \left(A \phi^{(1-4\xi)/\xi} + B \phi^{(1-4\xi)/2\xi} + C \right), \quad (2.16)$$

where

$$A := \frac{2\xi^2(p^2 - q^2)}{(4\xi - 1)^2}, \quad (2.17)$$

$$B := \frac{8q\xi^2(6\xi - 1)}{(4\xi - 1)^2}, \quad (2.18)$$

$$C := -\frac{\xi(6\xi - 1)(16\xi - 3)}{(4\xi - 1)^2}. \quad (2.19)$$

The constants in the solution (2.15) are given by

$$p^2 = \frac{(4\xi - 1)^2}{64\xi^4(6\xi - 1)^2} [32\xi^2(6\xi - 1)^2 A + (4\xi - 1)^2 B^2], \quad (2.20)$$

$$q = \frac{(4\xi - 1)^2 B}{8\xi^2(6\xi - 1)}. \quad (2.21)$$

The coupling constant ξ in the non-minimal coupling must be related to the constant C as (2.19). It is noted that the corresponding potential is quadratic for the de Sitter spacetime with the spatially flat slicing [14].

In the case of $\xi = 1/4$, the stealth solution is given by

$$\phi = K e^{w \sin H\tau}, \quad (2.22)$$

where K and w are constants. The corresponding potential is given by

$$V(\phi) = \frac{\Lambda}{3} \phi^2 \left[\bar{A} + \bar{B} \ln \phi - \frac{1}{2} (\ln \phi)^2 \right]. \quad (2.23)$$

The constants in the solution are given by

$$K = e^{\bar{B}+3/2}, \quad w^2 = 2\bar{A} - \bar{B} - \frac{3}{2}. \quad (2.24)$$

We assume the values of \bar{A} and \bar{B} such that $w^2 > 0$ holds.

1. Instanton

Here we shall impose regularity conditions for the scalar field at the de Sitter poles $H\tau = \pm\pi/2$ in order to find a regular instanton solution on de Sitter space. The conditions are $\phi'(\tau = \pm\pi/2H) = 0$. In the case of $\xi \neq 1/4$, they are satisfied in two cases; (i) $|q| > |p|$ and (ii) $|q| \leq |p|$ and $2\xi/(4\xi - 1)$ is an integer. In the case (i), the sign of ϕ remains unchanged and the configuration of the scalar field is asymmetric. In the case (ii), ϕ vanishes at some latitude. Hence ϕ changes the sign there if the integer is odd, while the sign is fixed for the even integer. In this case, the configuration of the scalar field is also asymmetric except for the case of $q = 0$.

If the scalar field is conformally coupled, i.e., $\xi = 1/6$, the solution is given by

$$\phi = \frac{1}{p \sin H\tau + q}. \quad (2.25)$$

Hence if $|q| > |p|$, the scalar field is well-defined on de Sitter space. The corresponding potential is very simple:

$$V(\phi) = \frac{\Lambda}{6} (p^2 - q^2) \phi^4 \quad (< 0). \quad (2.26)$$

In the case of $\xi = 1/4$, the scalar field (2.22) is regular at the coordinate boundary $\tau = \pm\pi/(2H)$. So it gives an instanton solution.

2. Creation of the universe

When we discuss creation of the universe by use of an instanton solution, the constraint on the solution becomes much more tight. The de Sitter universe can be quantum mechanically created at $\tau = 0$, where a' vanishes and then the spacetime can be analytically continued by the Wick rotation. Since the scalar field is also associated to the de Sitter spacetime, ϕ' also vanishes at $\tau = 0$. It is possible only in the case of $q = 0$. In addition, in order for the resulting scalar field in the de Sitter universe to be real, $2\xi/(4\xi - 1)$ must be an even integer, i.e.,

$$\frac{2\xi}{4\xi - 1} = 2n \quad \longleftrightarrow \quad \xi = \frac{n}{4n - 1}, \quad (2.27)$$

where n is a positive integer. The case of $\xi = 1/4$ is not possible to discuss the creation of the universe because ϕ' does not vanish at $\tau = 0$.

The scalar field and the corresponding potential are now given by

$$\phi = \phi_0 (\sin H\tau)^{2n}, \quad (2.28)$$

$$V(\phi) = \frac{\Lambda}{3} \phi^2 \left(A \phi^{-\frac{1}{n}} + C \right), \quad (2.29)$$

where

$$\phi_0 := p^{2n}, \quad (2.30)$$

$$A = 2p^2 n^2 \quad (> 0), \quad (2.31)$$

$$C = -\frac{n(2n+1)(4n+3)}{4n-1} \quad (< 0). \quad (2.32)$$

The domain of ϕ is given by $0 \leq \phi \leq \phi_0$. The first term in the potential dominates in the region of small ϕ where the potential is positive. Because of

$$V(\phi_0) = -\frac{\Lambda n(4n+1)\phi_0^2}{4n-1} < 0, \quad (2.33)$$

the potential has a negative domain.

Now let us discuss whether the present system also allows a de Sitter solution with a trivial scalar field, namely $\phi \equiv \phi_c = \text{constant}$. Equation (2.5) gives an algebraic equation for ϕ_c :

$$0 = \phi_c \left(\frac{6n}{4n-1} + \frac{2n-1}{2n} A \phi_c^{-\frac{1}{n}} + C \right), \quad (2.34)$$

and the corresponding energy-momentum tensor is given by

$$T_{\mu\nu} = -\frac{1}{3} \Lambda \phi_c^2 \left(A \phi_c^{-\frac{1}{n}} + C + \frac{3n}{4n-1} \right) g_{\mu\nu}. \quad (2.35)$$

Here we focus on the solution with $T_{\mu\nu} \equiv 0$ in order to compare with the stealth solution. Equations (2.34) and (2.35) show that such a solution is possible only for $\phi_c = 0$ and $n \geq 2$. In summary, in the theory with the potential (2.29) with $n \geq 2$, there are two de Sitter solutions with the same value of the cosmological constant; one is the de Sitter spacetime with a stealth scalar field and the other is one without such a non-trivial scalar field. In the next section, we will discuss which universe is preferred at the moment of creation.

III. CREATION PROBABILITY WITH A STEALTH SCALAR FIELD

We have seen that the theory with the potential (2.29) with $n \geq 2$ admits de Sitter solutions with or without a stealth scalar field. In this section, we discuss, by the use of the Hartle and Hawking's semi-classical instanton approach, which configuration is preferred at the moment of the universe creation.

In order to discuss the creation probability of the de Sitter universe, we evaluate the Euclidean action by use

of the semi-classical instanton solution $(g_{\mu\nu}^{(I)}, \phi_{(I)})$, which has been discussed in the previous section:

$$S_E = S_{g(E)} + S_{\phi(E)}, \quad (3.1)$$

where

$$S_{g(E)} = -\frac{1}{2} \int d^4x \sqrt{g^{(I)}} \left(R(g_{\mu\nu}^{(I)}) - 2\Lambda \right), \quad (3.2)$$

$$S_{\phi(E)} = \int d^4x \sqrt{g^{(I)}} \left[\frac{1}{2} \left(\frac{d\phi_{(I)}}{d\tau} \right)^2 + \frac{1}{2} \xi \phi_{(I)}^2 R(g_{\mu\nu}^{(I)}) + V(\phi_{(I)}) \right], \quad (3.3)$$

where $d^4x = d\tau d^3\mathbf{x}$. In the Hartle-Hawking proposal, the tunneling probability is proportional to $\exp(-S_E)$ [19]. The probability for the solution with $\phi \equiv 0$ is simply given by $\exp(-S_{g(E)}) (= \exp(24\pi^2/\Lambda))$. Therefore, the universe with the stealth scalar field is preferred if $S_{\phi(E)}(\phi_{\text{stealth}}) < 0$.

Putting the stealth solution (2.28) and $\xi = n/(4n-1)$, $S_{\phi(E)}$ is computed as

$$\begin{aligned} S_{\phi(E)}(\phi_{\text{stealth}}) &= 2\pi^2 \int_{-\pi/(2H)}^{\pi/(2H)} d\tau a^3 \left[\frac{1}{2} \left(\frac{d\phi_{\text{stealth}}}{d\tau} \right)^2 - \frac{n(2n+3)\Lambda}{3} \phi_{\text{stealth}}^2 + \frac{2p^2 n^2 \Lambda}{3} \phi_{\text{stealth}}^{(2n-1)/n} \right] \\ &= \frac{2\pi^2 p^{4n} n}{H} \int_{-\pi/(2H)}^{\pi/(2H)} d\tau (\cos H\tau)^3 (\sin H\tau)^{2(2n-1)} [4n - (4n+3)(\sin H\tau)^2] \\ &= \frac{24\pi^2 n}{(4n-1)(4n+1)(2n^2)^{2n}} \frac{A^{2n}}{\Lambda} > 0, \end{aligned} \quad (3.4)$$

where we used Eq. (2.31). Therefore, the universe with a stealth scalar field is not preferred at the moment of creation. However, as we will show below, the creation rate is almost the same as that of de Sitter with a trivial scalar field.

In order to show how plausible such a creation is, we shall evaluate $S_{\phi(E)}(\phi_{\text{stealth}})$. Since we discuss creation of the universe, we expect that the cosmological constant $\Lambda \sim O(1)$ and the coefficient in the potential $A \sim O(1)$ in the Planck unit. The prefactor in (3.4) is smaller than 0.00184 for $n \geq 2$ and vanishes rapidly as n increases. As a result, we can evaluate $e^{-S_{\phi(E)}} \gtrsim 0.998$, which means that the creation rate of the de Sitter universe with a stealth scalar field is almost the same as that without it.

IV. SUMMARY

In this paper, we have studied the contribution of a stealth scalar field to the creation probability of the de Sitter universe. With a certain form of the potential, we have constructed an exact stealth solution in the Hawking-Moss de Sitter instanton space. An analytic continuation of the instanton with a stealth scalar field to the de Sitter universe is allowed for a discrete value of the coupling constant.

Adopting the Hartle-Hawking proposal, we have shown that the creation probability with a stealth scalar field is less than that with a trivial scalar field, but both creation rates are almost the same for natural values of parameters. Actually, the creation probability with a stealth scalar field is larger if we adopt the Linde-Vilenkin pro-

posal because our result shows $S_{g(E)} + S_{\phi(E)}(\phi_{\text{stealth}}) < 0$ and hence $\exp(-|S_{g(E)} + S_{\phi(E)}|) = \exp(S_{g(E)} + S_{\phi(E)}) > \exp(S_{g(E)})$ holds. In both cases, since $|S_{\phi(E)}/S_{g(E)}| \ll 1$ is satisfied, the creation rate with a stealth scalar field is almost the same as that in vacuum and the effect of the stealth field is significant.

In the present paper, we have considered only the Hawking-Moss de Sitter instanton. However, one may expect that there exist other types of instantons. Examples are the Halliwell-Laflamme instanton for a conformally coupled scalar field ($\xi = 1/6$) without potential [20] and the Coleman-De Luccia instanton or Hawking-Turok instanton [21, 22] for a minimally coupled scalar field. If it is the case, we have to take into account such instantons as well, although we suspect that the highly symmetric de Sitter instanton is most preferable. We shall leave it as a future problem.

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